

Approximation of math model of the combined cutting soil's critical depth with influence of working speed

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Abstract. The purpose is approximation of mathematical model of processes of cutting and loosening of the soil for the receipt of analytical dependence of determination of critical depth of loosening taking into account the working speed at the combined tiered destruction of soil. An approximation based on regression analysis used in the processing of data derived from experiments with a number of parallel observations in the experiment. Built graph of the dependency between relative critical depth, working speed of the working body and angle cutting soil. Identify the value of critical depth loosening for five types of soil and full range of working speeds, depending on the physical and mechanical properties of the soil.

Keywords: angle cutting, depth loosening, operating speed, critical depth, free cutting, blocked cutting, combined cutting

INTRODUCTION

To eliminate the overcritical zone soil loosening and reduce the energy workflow [1 – 3, 22], need to know the critical depth of cutting depending on cut conditions (upper lower tier) and initial data for soil and speed of the working body [4 – 7]. Process of tiered combined cutting soil and mathematical model of determination the critical depth of loosening were examined and described in previous articles [8, 9, 20].



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To study the combined critical depth considering working speed was considered the destruction of soil unpin based on scheme (Fig.1), at the same time as the initial conditions were accepted the following assumptions:

- 1) soil – homogeneous isotropic medium characterized clutch, external and internal friction, density and moisture content;
- 2) the element of chip is viewing as a solid body in the form of a triangular prism with two symmetrical conical sectors on each sides;

3) the normal law of pressure distribution on the frontal plane of the working body in the chipping area is taken linear for the depth;

4) the critical depth is constant part of work of working body in landing mode or in steady mode.

Because of these researches was obtained mathematical model of the critical depth of cut in the combined tiered destruction of soil considering the working speed:

$$h_{кр.к} = \frac{-B - \sqrt{B^2 - 4AC}}{2A}, \quad (1)$$

where v , α_p , ψ – speed, cutting angle shear soil and cleaving soil angle; (look Fig.1); $v_{кр}$ – critical cutting speed at which changes the nature of the destruction of soil [9]; q_1 –

maximum soil pressure on the knife's surface in the upper tier by free-cutting process; $q_{кр}$ – critical value of pressure; b_k – width of the knife; $\gamma_{гр}$, c – specific gravity and friction coefficient of soil; g – acceleration of gravity; φ – angle of soil external friction; φ_0 – angle of soil internal friction; $k_{пер}$ – the ratio of the depth zone of guaranteed chipping soil ($k_{пер} = 0,9 \dots 0,95$); q_0 – the minimum value of pressure, acting on the surface; $h_B, h_{0л2}$ – depth of soil operation in a free and blocked cutting respectively; γ, γ_k – collapse zones' angles on combined cutting the soil in upper and lower tiers respectively [21]; ρ, δ, λ – angles formed by lateral chipping plane with the vertical plane.

$$\left\{ \begin{aligned} A &= (\text{ctg}\psi + \text{ctg}\alpha_p) c \left(\frac{\rho}{\sin\psi} + \frac{\cos\delta}{\cos\lambda} \right) - \frac{\sin(\alpha_p + \varphi + \varphi_0 + \psi)}{\cos\varphi \cos\varphi_0 \sin\alpha_p} b_k \left(1 + \frac{v}{v_{кр}} \right) \times \\ &\quad \times \frac{\gamma_{гр}}{g} \frac{\sin^2\alpha_p \cos\psi \cos\varphi}{\sin(\alpha_p + \psi) \sin(\alpha_p + \varphi)} v^2 \frac{\text{ctg}\gamma}{2b_k}; \\ B &= - \frac{\sin(\alpha_p + \varphi + \varphi_0 + \psi)}{\cos\varphi \cos\varphi_0 \sin\alpha_p} b_k \left(1 + \frac{v}{v_{кр}} \right) + \frac{\gamma_{гр}}{g} \frac{\sin^2\alpha_p \cos\psi \cos\varphi}{\sin(\alpha_p + \psi) \sin(\alpha_p + \varphi)} v^2 \times \\ &\quad \times \left(\frac{2 \cdot \text{ctg}\gamma_k \cdot h_B}{b_k} - \frac{h_B \cdot \text{ctg}\gamma}{b_k} + 1 \right) + \\ &\quad + (2h_B \text{ctg}\gamma_k + b_k) \frac{c}{\sin\psi} - 2c(\text{ctg}\psi + \text{ctg}\alpha_p) \left(\frac{\rho}{\sin\psi} + \frac{\cos\delta}{\cos\lambda} \right) h_B; \\ C &= \frac{\sin(\alpha_p + \varphi + \varphi_0 + \psi)}{\cos\varphi \cos\varphi_0 \sin\alpha_p} b_k \left(1 + \frac{v}{v_{кр}} \right) h_B \left(\frac{q_{кр} - q_1}{2} k_{пер} + \right. \\ &\quad \left. \frac{\gamma_{гр}}{g} \frac{\sin^2\alpha_p \cos\psi \cos\varphi}{\sin(\alpha_p + \psi) \sin(\alpha_p + \varphi)} v^2 \frac{2\text{ctg}\gamma_k h_B}{b_k} \right) - \\ &\quad - \left(2h_B^2 \text{ctg}\gamma_k + \frac{h_B^2 \text{ctg}\gamma}{2} \right) \frac{c}{\sin\psi} + c(\text{ctg}\psi + \text{ctg}\alpha_p) \left(\frac{\rho}{\sin\psi} + \frac{\cos\delta}{\cos\lambda} \right) h_B^2. \end{aligned} \right.$$

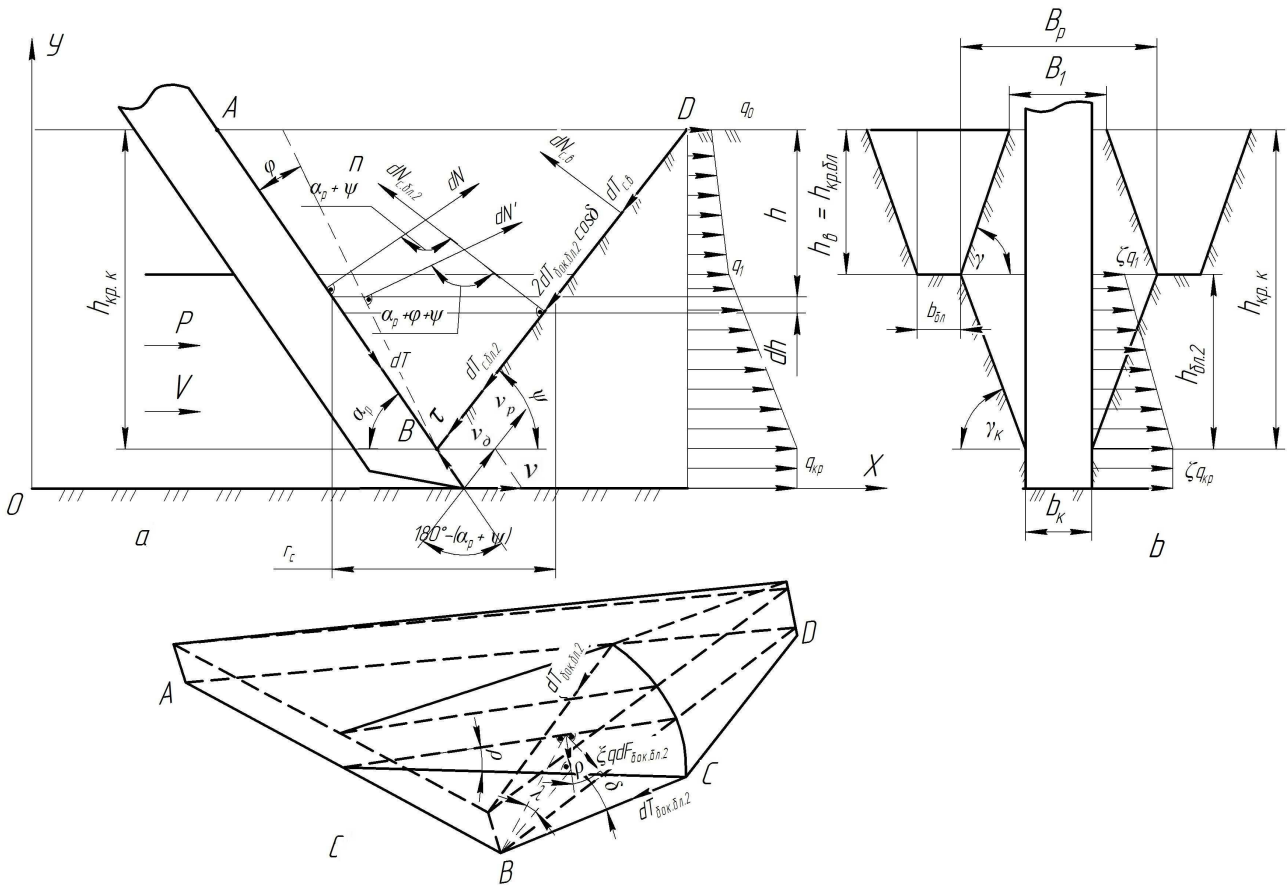


Fig.1. The scheme of interaction of a knife with soil environment about combined cutting soil: *a* – in the longitudinal plane; *b* – in the transverse plane; *c* – the form of chip`s element in the knife hollowing process

Calculated value of the relative depth of cleavage in view of working speed listed in the Table 1.

As the mathematical model is quite cumbersome and not easy to compute and further research is necessary to make an approximation of the model. For realization of approximating will use the regression multivariate analysis [11 – 13].

To determine the *b* – coefficient used regression analysis based on the method of least squares [18, 19].

Will write the equation of theoretical mathematical model of the critical depth of cut considering the speed in general view:

$$y = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_{1,2}x_1x_2 + b_{1,3}x_1x_3 + b_{2,3}x_2x_3 + b_{1,2,3}x_1x_2x_3, \quad (2)$$

where *b*₀ – free member;

*b*₁, *b*₂, *b*₃, *b*_{1,2}, *b*_{1,3}, *b*_{2,3}, *b*_{1,2,3} – factors that take into account linear impact on the interaction feedback factors of the first, second and third orders [10].

Draw the transfer of levels` factors natural values in the code dimensionless quantities in order to further build of standard matrix (Table 2):

$$x_k = \frac{X_k - X_{k_0}}{\Delta X_k}, \quad (3)$$

where *x*_{*k*} – coded values of *k*-factor; *X*_{*k*} – natural current value of *k*-factor; *X*_{*k*0} – initial

(zero) level $X_{k_0} = \frac{X_{\max} + X_{\min}}{2}$ – factor;

ΔX_k – the interval of variation of *k*-factor.

the interval of variation of *k*-factor [14 – 16].

After the coding level factors take values: "1" – the upper level; "-1" – lower level; "0" – zero level. As zero level, take the center of spacing, which conducted the study (Table 3).

Table 1. Value of relative critical depth

Semi-solid loam						
$\nu, \frac{m}{s}$	0	1	2	3	4	5,18
α_p, \circ						
20	8,90	10,32	11,44	12,69	14,13	16,24
30	7,16	9,02	10,44	11,40	14,07	17,13
40	5,78	7,86	9,43	11,29	13,70	17,69
50	4,58	6,63	8,25	10,20	12,75	17,27
60	3,58	5,47	7,00	8,86	11,33	15,69
Longing clay						
$\nu, \frac{m}{s}$	0	1	2	3	4	5,55
α_p, \circ						
20	8,28	10,80	11,56	12,44	13,47	15,45
30	6,83	9,16	10,20	11,44	12,98	16,33
40	5,59	7,79	9,05	10,58	12,53	17,10
50	4,43	6,39	7,73	9,39	11,58	16,95
60	3,33	5,13	6,60	8,44	10,94	17,64
Solid sandy loam						
$\nu, \frac{m}{s}$	0	1	2	3	4	4,45
α_p, \circ						
20	12,28	14,13	16,00	18,30	21,25	22,99
30	9,73	12,10	14,42	17,50	21,99	24,72
40	7,73	10,47	13,06	16,58	22,15	25,88
50	6,03	8,72	11,37	15,02	20,93	25,07
60	4,53	6,94	9,29	12,64	18,21	22,19

Regression coefficients calculated by the formula:

$$b_0 = \frac{1}{n} \sum_{x=i}^n y_i, \quad (4)$$

$$b_i = \frac{1}{n} \sum_{i=1}^n x_i y_i, \quad (5)$$

$$b_{iu} = \frac{1}{n} \sum_{i=1}^n x_i x_u y_i, \quad (6)$$

$$b_{iuk} = \frac{1}{n} \sum_{i=1}^n x_i x_u x_k y_i, \quad (7)$$

where x_i, x_u, x_k – natural values i, u, k – factor experiments y_i – parameter optimization feedback.

Perform the calculations of approximated value of critical cutting depth:

$$h_{kp} = b_0 + \frac{b_i}{\Delta X_i} (X_i - X_{i0}) + \frac{b_{iu}}{\Delta X_i \Delta X_u} (X_i - X_{i0})(X_u - X_{u0}) + \frac{b_{iuk}}{\Delta X_i \Delta X_u \Delta X_k} (X_i - X_{i0}) \times (X_u - X_{u0})(X_k - X_{k0}); \quad (8)$$

When substituting numerical values of the components of the equation (8) for different soil types, we obtain the following expression:

Table 2. Value factors and levels of variation for different types of soil

Factor	Marking		Interspace (loam)	Interspace (sandy loam)	Interspace (clay)	Level of variation									Code		
	Natural	Code				Natural											
						Semisolid loam			Solid sandy loam			Hardplastic clay					
						top	mid.	bot.	top	mid.	bot.	top	mid.	bot.	top	mid.	bot.
Blade width, b, m	X_1	x_1	0,1	0,1	0,1	0,25	0,15	0,05	0,25	0,15	0,05	0,25	0,15	0,05	1	0	-1
Knife's cutting angle, α, rad	X_2	x_2	0,349	0,349	0,349	1,047	0,698	0,349	1,047	0,698	0,349	1,047	0,698	0,349	1	0	-1
Cutting speed, $V, m/s$	X_3	x_3	2,09	1,725	2,275	5,18	3,09	1	4,45	2,725	1	5,55	3,275	1	1	0	-1

Table 3. Matrix of 3-factor`s planning

№	x_0	x_1	x_2	x_3	$x_1 x_2$	$x_1 x_3$	$x_2 x_3$	$x_1 x_2 x_3$	y_i Semisolid loam	y_i Solid sandy loam	y_i Hardplastic clay
1	+	-1	-1	-1	1	1	1	-1	0,516	0,707	0,54
2	+	1	-1	-1	-1	-1	1	1	2,581	3,533	2,702
3	+	-1	1	-1	-1	1	-1	1	0,274	0,347	0,257
4	+	1	1	-1	1	-1	-1	-1	1,369	1,737	1,285
5	+	-1	-1	1	1	1	-1	1	0,812	1,15	0,773
6	+	1	-1	1	-1	-1	-1	-1	4,061	5,748	3,864
7	+	-1	1	1	-1	1	1	-1	0,785	1,11	0,882
8	+	1	1	1	1	-1	1	1	3,925	5,548	4,411
Average value of feedback y_i									1,79	2,485	1,839

(for semisolid loam):

$$\begin{aligned}
 h_{kp} = & 1,79 + 11,91(b - 0,15) - 0,5788 \div \\
 & \times (\alpha_p - 0,689) + 0,2894(v - 3,09) - \\
 & - 3,8682(b - 0,15)(\alpha_p - 0,689) + \\
 & + 1,933(b - 0,15)(v - 3,09) + 0,2207 \times \\
 & \times (\alpha_p - 0,689)(v - 3,09) + \\
 & + 1,4806(b - 0,15)(\alpha_p - 0,689)(v - 3,09);
 \end{aligned}
 \tag{9}$$

(for solid sandy loam):

$$\begin{aligned}
 h_{kp} = & 2,485 + 16,57(b - 0,15) - 0,8596 \times \\
 & \times (\alpha_p - 0,689) + 0,524(v - 2,725) - \\
 & - 5,7304(b - 0,15)(\alpha_p - 0,689) + \\
 & + 3,4956(b - 0,15)(v - 2,725) + 0,3986 \times \\
 & \times (\alpha_p - 0,689)(v - 2,725) + 2,6577 \times \\
 & \times (b - 0,15)(\alpha_p - 0,689)(v - 2,725)
 \end{aligned}
 \tag{10}$$

(for hardplastic clay):

$$\begin{aligned}
 h_{kp} = & 1,839 + 12,26(b - 0,15) - 0,3752 \times \\
 & \times (\alpha_p - 0,689) + 0,2826(v - 3,275) - \\
 & - 2,4928(b - 0,15)(\alpha_p - 0,689) + \\
 & + 1,8857(b - 0,15)(v - 3,275) + 0,3715 \times \\
 & \times (\alpha_p - 0,689)(v - 3,275) + 2,4811 \times \\
 & \times (b - 0,15)(\alpha_p - 0,689)(v - 3,275)
 \end{aligned}
 \tag{11}$$

Puts the obtained coefficients of equations, theoretical and approximated value of critical cutting depth in Table 4 and 5.

Table 4. The coefficients of the regression equation b_i and approximated value of combined cutting critical depths

	Loam	Sandy loam	Clay
b_0	1,790	2,485	1,839
b_1	1,194	1,657	1,226
b_2	-0,202	-0,300	-0,131
b_3	0,605	0,904	0,643
b_{12}	-0,135	-0,200	-0,087
b_{13}	0,404	0,603	0,429
b_{23}	0,161	0,240	0,295
b_{123}	0,108	0,160	0,197

Carry the appreciation of the regression coefficients and adequacy test of regression equations using dispersion analysis.

Determine the degree of deviation values u – research of a random variable from its average value:

$$S_u^2 = \sum_{i=1}^n (y_i - \bar{y})^2, \tag{12}$$

where \bar{y} – average value of the research.

Dispersion of the research:

$$S_y^2 = \frac{S_u^2}{N}, \tag{13}$$

where N – numbers of researches.

The statistical significance of the regression coefficients is checked using t – Student's criterion [14 – 17]:

$$t_{bi} = \frac{|b_i|}{S_{bi}}, \tag{14}$$

Table 5. Plan 2³ and the results of calculations

№	Blade width, m	Cutting angle, rad	Cutting speed, m/s			Teor. values of critical depth, m			Teor. values of critical depth, m			Teor. values of critical depth, m		
			Loam	Sandy loam	Clay	Loam			Sandy loam			Clay		
1	0,05	0,349	1,0	1,0	1,0	0,516	0,5156	0,03	0,707	0,707	0,3	0,54	0,54	0,04
2	0,25	0,349	1,0	1,0	1,0	2,581	2,5806	0,03	3,533	3,533	0,8	2,702	2,702	0,04
3	0,05	1,047	1,0	1,0	1,0	0,274	0,2736	0,03	0,347	0,347	0,55	0,257	0,257	0,011
4	0,25	1,047	1,0	1,0	1,0	1,369	1,3686	0,03	1,737	1,737	0,44	1,285	1,285	0,04
5	0,05	0,349	5,18	4,45	5,55	0,812	0,8116	0,03	1,15	1,15	0,88	0,773	0,773	0,011
6	0,25	0,349	5,18	4,45	5,55	4,061	4,0606	0,03	5,748	5,748	0,88	3,864	3,864	0,044
7	0,05	1,047	5,18	4,45	5,55	0,785	0,7846	0,03	1,11	1,11	0,66	0,882	0,882	0,011
8	0,25	1,047	5,18	4,45	5,55	3,925	3,9246	0,03	5,548	5,548	0	4,411	4,411	0

$$S_{bi}^2 = \frac{S_y^2}{N}, \quad (15)$$

Defined by the formula (11) value t_{bi} is compared with tabulated values t_m [13,

Tab.3.2.]. When $t_{bi} > t_m$, the coefficient of the regression equation is significant. Otherwise, this ratio should be excluded from the equation.

Table 6. Research dispersion, the statistical significance of the regression coefficients, quadratic error of regression coefficients

№	Semisolid loam				Solid sandy loam				Hardplastic clay						
	S_u^2	S_y^2	S_{bi}	t_{bi}	S_u^2	S_y^2	S_{bi}	t_{bi}	S_u^2	S_y^2	S_{bi}	t_{bi}			
1	1,623	2,051	0,506	3,535	4,137	0,719	3,456	1,688	2,251	0,530	3,467				
2	0,626			2,357								1,098	2,304	0,744	2,312
3	2,298			0,399								4,571	0,417	2,504	0,246
4	0,177			1,196								0,560	1,257	0,307	1,213
5	0,956			0,266								1,782	0,277	1,137	0,164
6	5,157			0,797								10,647	0,838	4,100	0,808
7	1,010			0,319								1,891	0,333	0,916	0,555
8	4,558			0,213								9,382	0,222	6,614	0,370

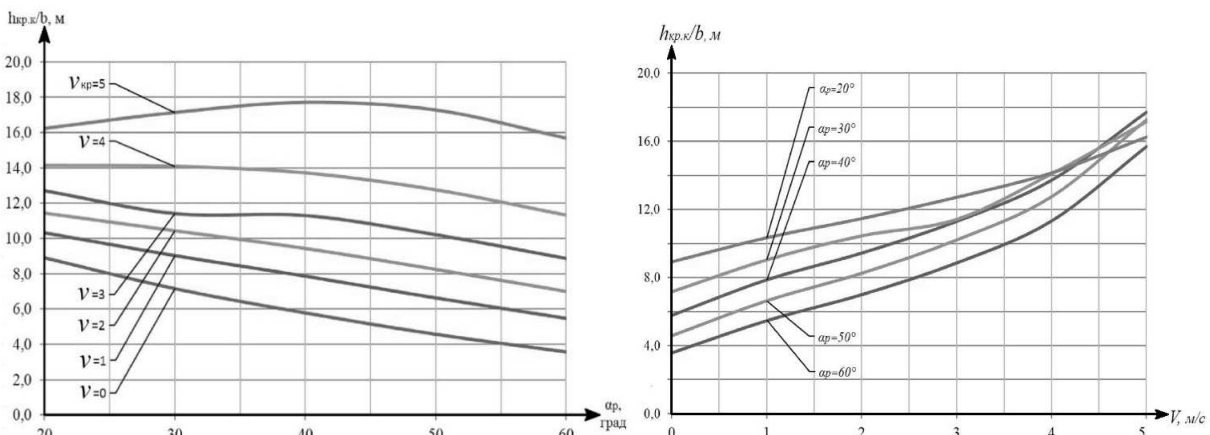


Fig.2. Dependence of the approximated relative critical depth combined cutting soil $h_{kp.k.}$ from the speed by changing the angle of cutting for semisolid loam

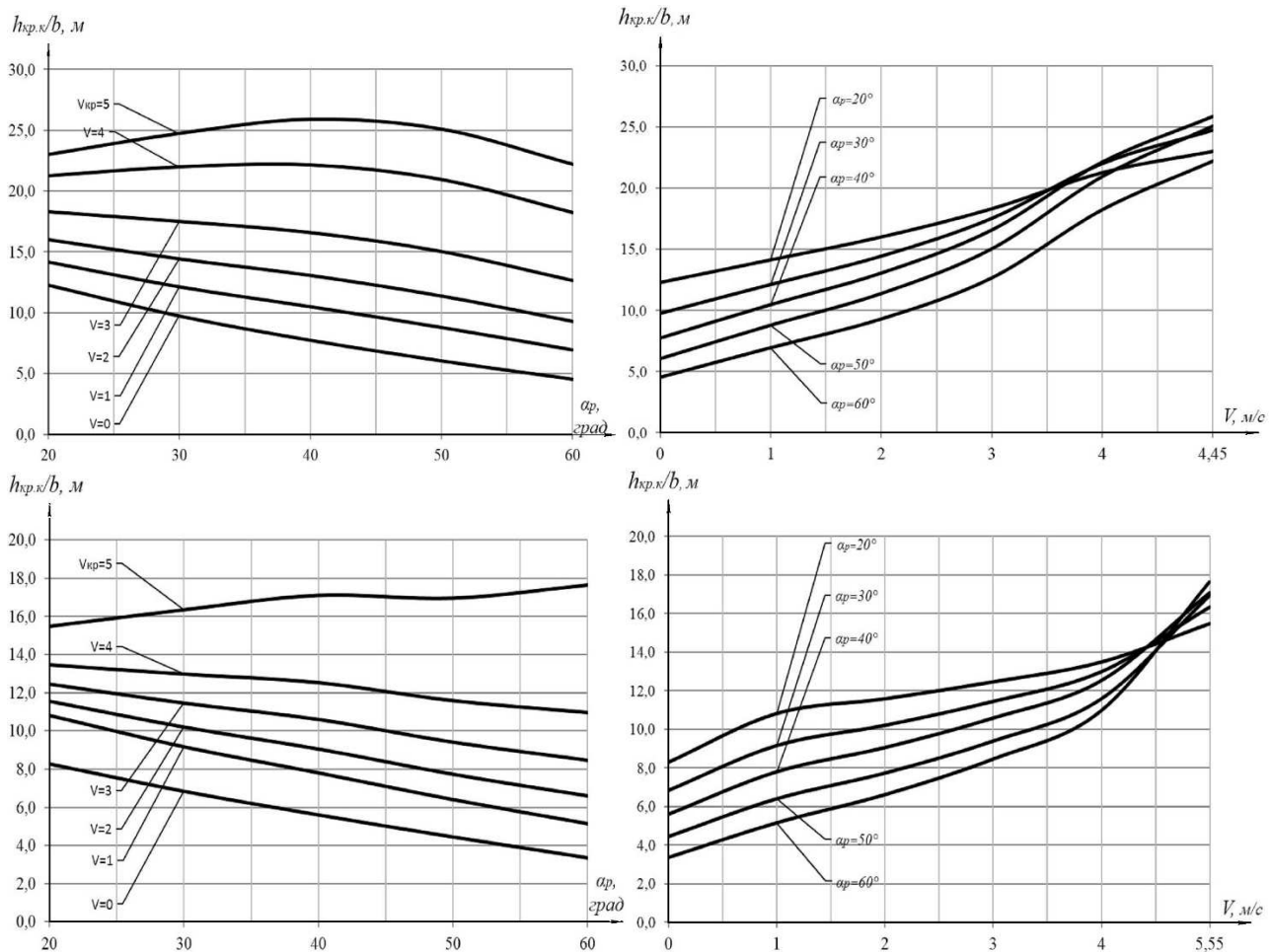


Fig.3. Dependence of the approximated relative critical depth combined cutting soil $h_{кр.к.}$ from the speed by changing the angle of cutting and dependence of approximated relative critical depth combined cutting soil $h_{кр.к.}$ from the angle when changing cutting speed, solid sandy loam and hardplastic clay

The value of the research dispersion, statistical significance of regression coefficients and quadratic error of the regression coefficients are entered in Table 6. Dependence of the approximated relative critical depth combined cutting soil $h_{кр.к.}$ from the speed for different types of soil are shown on Fig. 2 and Fig. 3.

CONCLUSIONS

1. The possibility of regression analysis applying of analytical model to approximate the critical depths of combined cutting soil has been shown.
2. Critical depth of combined cutting soil is approximated by the equation with a combination of factors like (8...15), approximation error is less than 1%.

3. Under the calculation dependence, increasing operating speed of 0 m/s to 4 m/s approximated value of relative critical depth of combined cutting $h_{кр.к.}$ increases from 3,58 to 14,13 meters for semi-loam; from 4,53 to 22,15 for solid sandy loam; from 3,33 to 13,47 for hardplastic clay depending on the cutting angle.

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Аппроксимация математической модели критической глубины комбинированного резания грунтов на основе регрессионного анализа

Святослав Кравец, Роман Зоря

Аннотация. Представлены результаты аппроксимации математической модели процессов резания и рыхления почвы для получения аналитической зависимости определения критической глубины разрыхления с учетом скорости рабочего органа при комбинированном поперечном разрушении почвы. Проведено аппроксимацию на основе регрессионного анализа, используемую при обработке данных, полученных в результате экспериментов с определенным количеством параллельных наблюдений в опыте. Построены графики зависимости между аппроксимированной относительной критической глубиной, рабочей скоростью рабочего органа и углом резки почвы. Определены значения критической глубины разрыхления для пяти типов грунта и полного диапазона рабочих скоростей, в зависимости от физико-механических свойств почвы.

Ключевые слова: угол резания, глубина разрыхления, рабочая скорость, критическая глубина, комбинированная резка, уравнение регрессии, дисперсия исследования.