Problem of plane strain state of two-layer body in dynamic elastic-plastic formulation (Part II)

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Abstract. Composite materials are widely used in industry and everyday life. Mathematical modeling of composite materials began to be actively developed in the 50s and 60s of the last century. Composite materials began to be actively used in industry only at the end of the 70s of the last century. From that time to this day, interest in composite materials has not weakened, and the demands of modern industry and production are constantly increasing. The areas and branches of application of composite materials are expanding. Many different methods are used to calculate and develop composite materials. This article is part two of the previous article, where there is an investigation of the contact problem of the interaction of a striker with a two-layers composite base in a dynamic elastic-plastic mathematical formulation. This composite base is rigidly attached to an absolutely hard half-space. Its first (top) layer is made of steel, and the second (bottom) layer is made of glass. Glass is a widely available cheap amorphous material, the properties of which cannot be degraded as an result of aging, corrosion, and creep processes. The glass layer can be strengthened by reinforcement and hardening. Therefore, composite materials made on the basis of glass are important in modern production; their use gives a great economic benefit. Rigid adhesion of the layers to each other is assumed. The impact process was modelled as a non-stationary plane strain state problem with an even distributed load in the contact area, which changes according to a linear law. The fields of the Odquist parameter and normal stresses were studied depending on the size of the contact area. In this article as in part I for the design of composite and reinforced material the non-stationary contact problem of plane strain state has been solved in more precise elastic-plastic mathematical formulation. To consider the physical nonlinearity of the deformation process, the method of successive approximations is used, which makes it possible to reduce the nonlinear problem to a solution of the sequences of linear problems.

In contrast to the previous article (Part I), in this papers there is an investigation of the strain-stress state, the fields of the Odquist parameter and normal stresses depending on the thickness of the first (upper) steel layer.

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INTRODUCTION

Glass is a very strong and very fragile material at the same time. The fragility of glass is due to the fact that there are many microcracks on the surface, and when a load is applied to the glass surface, these microcracks begin to grow and lead to the destruction of glass products. If we glue or immobilize the tops of microcracks on the surface, we will get a strong reinforced armed material that will be lighter, stronger and not subject to degradation of material properties such as aging, corrosion and creep. The upper reinforcing layer of metal can be applied to the glass surface by sputtering so that the metal atoms of the steel penetrate deeply, fill microcracks and bind their tops. The top layer can be quite thin.

Glass is also convenient in that it can be poured into the frame of the reinforcement and thus can be further strengthened. As reinforcing elements, metal wire, polysilicate, polymer, polycarbon compounds, which can have a fairly small thickness, can be used. The thickness of such reinforcing materials can be equal to the thickness of several atomic layers, such as graphene.

In [1, 2], a new approach to solving the problems of impact and nonstationary interaction in the elastoplastic mathematical formulation was developed. In this papers like in non-stationary problems [3, 4], the action of the striker is replaced by a distributed load in the contact area, which changes according to a linear law. The contact area remains constant. The developed elastoplastic formulation makes it possible to solve impact problems when the dynamic change in the boundary of the contact area is considered and based on this the movement of the striker as a solid body with a change in the penetration speed is taken into account. Also, such an elastoplastic formulation makes it possible to consider the hardening of the material in the process of nonstationary and impact interaction.

The solution of problems for composite cylindrical shells [5], elastic half-space [6], elastic layer [7], elastic rod [8, 9] were developed using method of the influence functions [10].

In contrast from the work [11], in this paper, we investigate the impact process of hard body with plane area of its surface on the top of the composite beam which consists first thin metal layer and second main glass layer. In contrast from the work [12], the fields of plastic deformations and, stresses were determined relative to the thickness of the first top layer of composite base.

PROBLEM FORMULATION AND SOLUTION ALGORITHM

Deformations and their increments [1 – 4], Odquist parameter, effective and principal stresses are obtained from the numerical solution of the dynamic elastic-plastic interaction problem of infinite composite beam $\{-L/2 \le x \le L/2;\quad 0 \le y \le B; -\infty \le z \le \infty\}$ in the plane of its cross section in the form of rectangle. It is assumed that the stress-strain state in each cross section of the cylinder is the same, close to the plane deformation, and therefore it is necessary to solve the equation for only one section in the form of a rectangle $\Sigma = L \times B$ with two layers: first steel layer $\{-L/2 \le x \le L/2; \quad -\infty \le z \le \infty;$ $B-h \le y \le B\}$ and second glass layer $\{-L/2 \le x \le L/2;\ 0 \le y \le B-h; -\infty \le z \le \infty\}$ a notch-crack with length l along the segment and contacts absolute hard half-space $\{y \le 0\}$. We assume that the contact between the lower surface of the first metal layer and the upper surface of the second glass layer is ideally rigid.

From above on a body the absolutely rigid drummer contacting along a segment $\{|x| \le A;$ $y = B\}$. Its action is replaced by an even distributed stress $-P$ in the contact region, which changes over time as a linear function $P = p_{01} + p_{02}t$. Given the symmetry of the deformation process relative to the line $x = 0$, only the right part of the cross section is considered below (Fig. 1).

The equations of the plane dynamic theory are considered, for which the components of the displacement vector $\mathbf{u} = (u_x, u_y)$ are related to the components of the strain tensor by Cauchy relations:

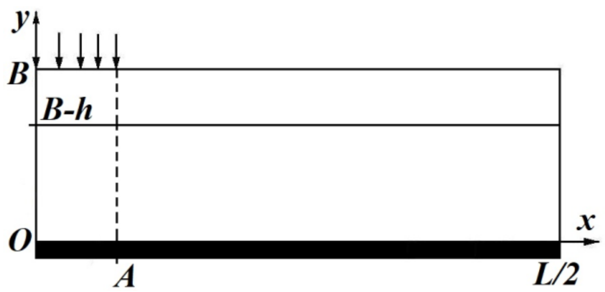

Fig. 1. Geometric scheme of the problem

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x}, \quad \varepsilon_{yy} = \frac{\partial u_y}{\partial y}, \quad \varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right).$$

The equations of motion of the medium have the form:

$$\begin{aligned} &\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = \rho \frac{\partial^2 u_x}{\partial t^2}, \\ &\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = \rho \frac{\partial^2 u_y}{\partial t^2}, \end{aligned} \tag{1}$$

where ρ − material density.

The boundary and initial conditions of the problem have the form:

$$\begin{aligned} &x = 0,\ 0 < y < B: u_x = 0,\ \sigma_{xy} = 0, \\ &x = L/2, 0 < y < B: \sigma_{xx} = 0,\ \sigma_{xy} = 0, \\ &y = 0,\ 0 < x < L/2:\ u_y = 0,\ \sigma_{xy} = 0, \\ &y = B,\ 0 < x < A:\ \sigma_{yy} = -P,\ \sigma_{xy} = 0, \\ &y = B,\ A < x < L/2:\ \sigma_{yy} = 0,\ \sigma_{xy} = 0. \end{aligned} \tag{2}$$

$$\begin{aligned} &u_x\big|_{t=0} = 0,\ u_y\big|_{t=0} = 0,\ u_z\big|_{t=0} = 0, \\ &\dot{u}_x\big|_{t=0} = 0,\ \dot{u}_y\big|_{t=0} = 0,\ \dot{u}_z\big|_{t=0} = 0. \end{aligned} \tag{3}$$

The determinant relations of the mechanical model are based on the theory of non-isothermal plastic flow of the medium with hardening under the condition of Huber-Mises fluidity. The effects of creep and thermal expansion are neglected. Then, considering the components of the strain tensor by the sum of its elastic and plastic components [13, 14], we obtain expression for them:

$$\begin{aligned} &\varepsilon_{ij} = \varepsilon_{ij}^e + \varepsilon_{ij}^p,\ d\varepsilon_{ij}^p = s_{ij} d\lambda, \\ &\varepsilon_{ij}^e = \frac{1}{2G} s_{ij} + K\sigma + \varphi. \end{aligned} \tag{4}$$

here $s_{ij} = \sigma_{ij} - \delta_{ij}\sigma$ – stress tensor deviator; δ_{ij} – Kronecker symbol; E – modulus of elasticity (Young's modulus); G – shear modulus; $K_1 = (1 - 2\nu)/(3E)$, $K = 3K_1$ – volumetric compression modulus, which binds in the ratio $\varepsilon = K\sigma + \phi$ volumetric expansion 3ε (thermal expansion $\phi \equiv 0$); $\sigma = (\sigma_{xx} + \sigma_{yy} + \sigma_{zz})/3$ − mean stress; $d\lambda$ – some scalar function [15], which is determined by the shape of the load surface and we assume that this scalar function is quadratic function of the stress deviator s_{ij} [13 – 15].

$$d\lambda = \begin{cases} 0 \quad (f \equiv \sigma_i^2 - \sigma_S^2(T) < 0) \\ \dfrac{3d\varepsilon_i^p}{2\sigma_i} \quad (f = 0, df = 0) \\ (f > 0 - \text{inadmissible}) \end{cases}, \tag{5}$$

$$d\varepsilon_i^p = \frac{\sqrt{2}}{3}\Big(\left(d\varepsilon_{xx}^p - d\varepsilon_{yy}^p\right)^2 + \left(d\varepsilon_{xx}^p - d\varepsilon_{zz}^p\right)^2 + \left(d\varepsilon_{yy}^p - d\varepsilon_{zz}^p\right)^2 + 6\left(d\varepsilon_{xy}^p\right)^2\Big)^{1/2},$$

$$\sigma_i = \frac{1}{\sqrt{2}}\Big(\left(\sigma_{xx} - \sigma_{yy}\right)^2 + \left(\sigma_{xx} - \sigma_{zz}\right)^2 + \left(\sigma_{yy} - \sigma_{zz}\right)^2 + 6\sigma_{xy}^2\Big)^{1/2}.$$

The material is strengthened with a hardening factor $\eta*$ [1 – 4]:

$$\begin{aligned} &\sigma_S(T) = \sigma_{02}(T_0)\left(1 + \frac{\kappa(T)}{\varepsilon_0}\right)^{\eta*}, \\ &\varepsilon_0 = \frac{\sigma_{02}(T_0)}{E}, \end{aligned} \tag{6}$$

where T – temperature; κ – Odquist parameter, $T_0 = 20^\circ C$, $\eta*$ – hardening coefficient; $\sigma_S(T)$ – yield strength after hardening of the material at temperature T.

Rewrite (4) in expanded form:

$$d\varepsilon_{xx} = d\left(\frac{\sigma_{xx}-\sigma}{2G}+K\sigma\right)+(\sigma_{xx}-\sigma)d\lambda,$$
$$d\varepsilon_{yy} = d\left(\frac{\sigma_{yy}-\sigma}{2G}+K\sigma\right)+(\sigma_{yy}-\sigma)d\lambda,$$
$$d\varepsilon_{zz} = d\left(\frac{\sigma_{zz}-\sigma}{2G}+K\sigma\right)+(\sigma_{zz}-\sigma)d\lambda, \qquad (7)$$
$$d\varepsilon_{xy} = d\left(\frac{\sigma_{xy}}{2G}\right)+\sigma_{xy}d\lambda,$$

In contrast to the traditional plane deformation, when $\Delta\varepsilon_{zz}(x,y)=\text{const}$, for a refined description of the deformation of the specimen, taking into account the possible increase in longitudinal elongation $\Delta\varepsilon_{zz}$, we present in its form [2 – 4, 16]:

$$\Delta\varepsilon_{zz}(x,y)=\Delta\varepsilon_{zz}^{0}+\Delta\chi_x x+\Delta\chi_y y, \qquad (8)$$

where unknown $\Delta\chi_x$ and $\Delta\chi_y$ describe the bending of the prismatic body (which simulates the plane strain state in the solid mechanics) in the *Ozx* and *Ozy* planes, respectively, and $\Delta\varepsilon_{zz}^{0}$ – the increments according to the detected deformation bending along the fibers $x=y=0$.

The solution algorithm is the same as in [12].

NUMERICAL SOLUTION

The explicit scheme of the finite difference method was used with a variable partitioning step along the axes *Ox* (*M* elements) and *Oy* (*N* elements). The step between the split points was the smallest in the area of the layers contact and at the boundaries of the computational domain. Since the interaction process is fleeting, this did not affect the accuracy in the first thin layer, areas near the boundaries, and the adequacy of the contact interaction modelling.

The use of finite differences [17] with variable partition step for wave equations is justified in [18], and the accuracy of calculations with an error of no more than $O\left((\Delta x)^2+(\Delta y)^2+(\Delta t)^2\right)$ where Δx, Δy and Δt – increments of variables: spatial *x* and *y* and time *t*. A low rate of change in the size of the steps of the partition mesh was ensured. The time step was constant.

The resolving system of linear algebraic equations with a banded symmetric matrix was solved by the Gauss method according to the Cholesky scheme.

In [19], during experiments, compact samples were destroyed in 21 – 23 ms. The process of destruction of compact specimens from a material of size and with contact loading as in [19] was modelled in a dynamic elastoplastic formulation, considering the unloading of the material and the growth of a crack according to the local criterion of brittle fracture. The samples were destroyed in 23 ms. This confirms the correctness and adequacy of the developed formulation and model.

Figs. 2 – 29 show the results of calculations of two layers specimens with a hardening factor of the material $\eta^*=0,05$. The first high layer has made from hard steel and its thickness was equal $h=h_1=0.1$ mm, $h=h_2=0.3$ mm and $h=h_3=0.5$ mm. The second main low layer has made from quartz glass. Contact between two layers is an ideal. Calculations were made at the following parameter values: temperature $T=50\ ^{\circ}\text{C}$; the size of the contact zone $a=2A=0.5$ mm; $L=60$ mm; $B=10$ mm; $\Delta t=3.21\cdot10^{-8}$ s; $p_{01}=8$ MPa; $p_{02}=10$ MPa; $M=62$; $N=100$. The smallest splitting step was 0,005 mm, and the largest 2,6 mm ($\Delta x_{\min}=0,005$ mm; $\Delta y_{\min}=0,01$ mm (only the first layer); $\Delta x_{\max}=2,6$ mm; $\Delta y_{\max}=0,65$ mm).

Figs. 2 – 4, 11 – 13, 20 – 22 show the fields of the Odquist parameter κ, normal stresses σ_{xx} and σ_{yy} respectively at the time $t_1=2.57\cdot10^{-6}$ s. The fields of Odquist parameter κ, normal stresses σ_{xx} and σ_{yy} respectively are at the Figs. 5 – 7, 14 – 16, 23 – 25 at the time $t_2=3.82\cdot10^{-6}$ s. Figs. 8 – 10, 17 – 19, 26 – 28 show the fields of κ, σ_{xx} and σ_{yy} respectively at the time $t_3=5.13\cdot10^{-6}$ s.

Fig. 2. Odquist parameter κ when $h = h_1$, $t = t_1$

Fig. 3. Odquist parameter κ when $h = h_2$, $t = t_1$

Fig. 4. Odquist parameter κ when $h = h_3$, $t = t_1$

Fig. 5. Odquist parameter κ when $h = h_1$, $t = t_2$

Fig. 6. Odquist parameter κ when $h = h_2$, $t = t_2$

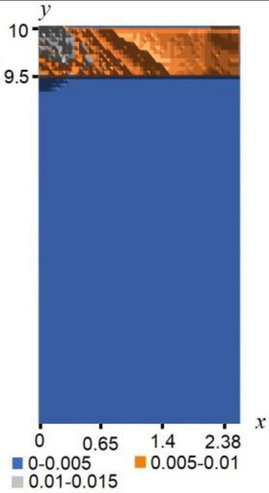

Fig. 7. Odquist parameter κ when $h = h_3$, $t = t_2$

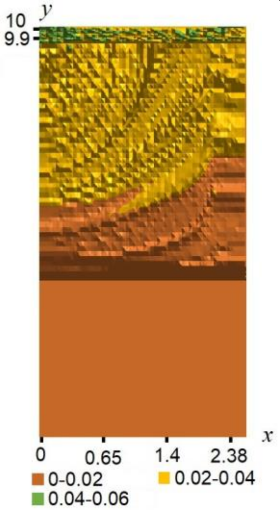

Fig. 8. Odquist parameter κ when $h = h_1$, $t = t_3$

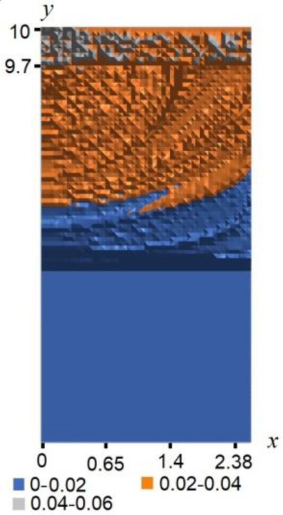

Fig. 9. Odquist parameter κ $h = h_2$, $t = t_3$

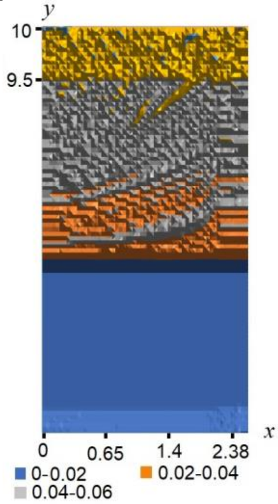

Fig. 10. Odquist parameter κ $h = h_3$, $t = t_3$

Fig. 11. Stress σ_{xx} when $h = h_1$, $t = t_1$

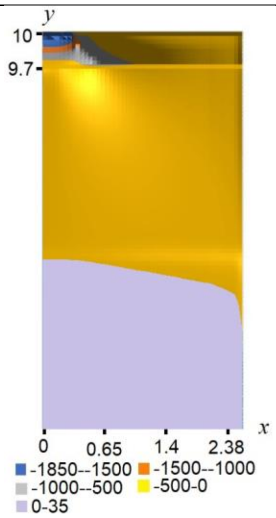

Fig. 12. Stress σ_{xx} when $h = h_2$, $t = t_1$

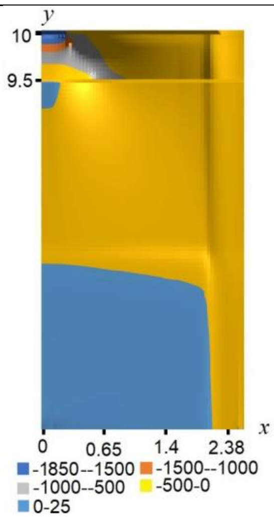

Fig. 13. Stress σ_{xx} when $h = h_3$, $t = t_1$

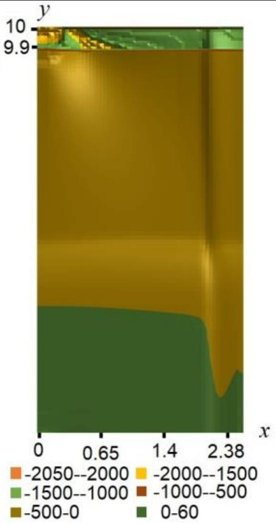

Fig. 14. Stress σ_{xx} when $h = h_1$, $t = t_2$

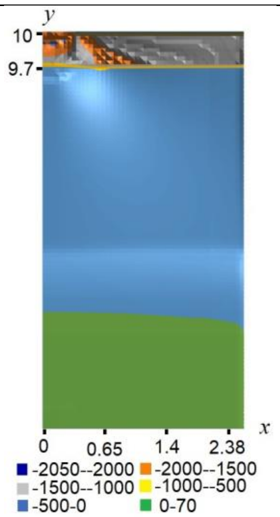

Fig. 15. Stress σ_{xx} when $h = h_2$, $t = t_2$

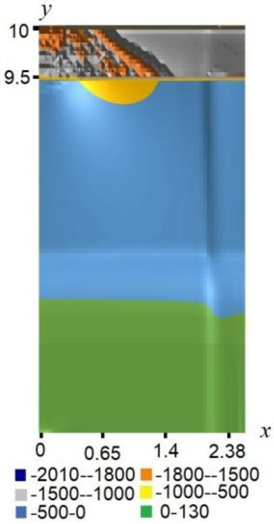

Fig. 16. Stress σ_{xx} when $h = h_3$, $t = t_2$

Fig. 17. Stress σ_{xx} when $h = h_1$, $t = t_3$

Fig. 18. Stress σ_{xx} when $h = h_2$, $t = t_3$

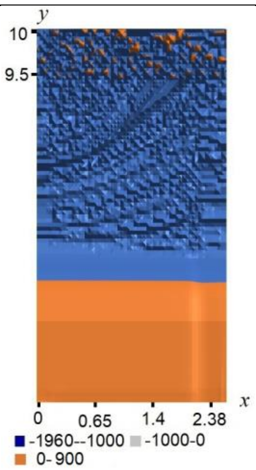

Fig. 19. Stress σ_{xx} when $h = h_3$, $t = t_3$

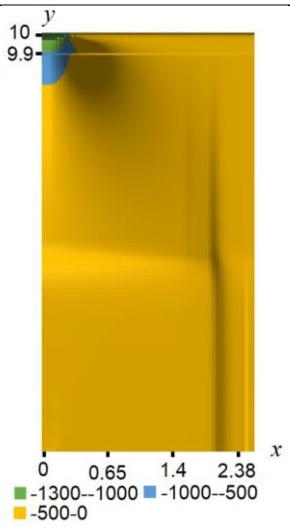

Fig. 20. Stress σ_{yy} when $h = h_1$, $t = t_1$

Fig. 21. Stress σ_{yy} when $h = h_2$, $t = t_1$

Fig. 22. Stress σ_{yy} when $h = h_3$, $t = t_1$

Fig. 23. Stress σ_{yy} when $h = h_1$, $t = t_2$

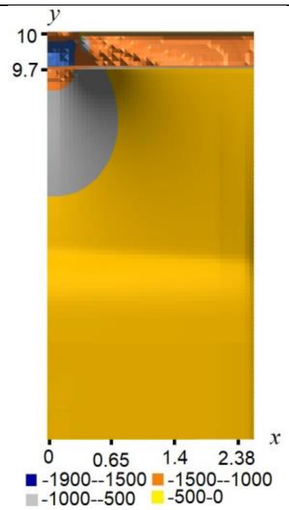

Fig. 24. Stress σ_{yy} when $h = h_2$, $t = t_2$

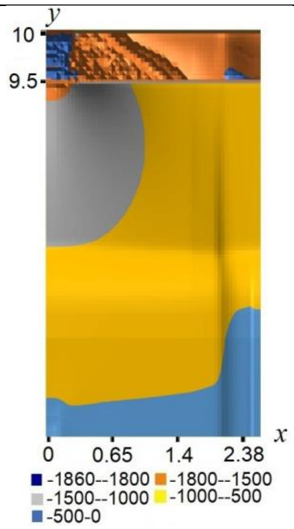

Fig. 25. Stress σ_{yy} when $h = h_3$, $t = t_2$

Fig. 26. Stress σ_{yy} when $h = h_1$, $t = t_3$

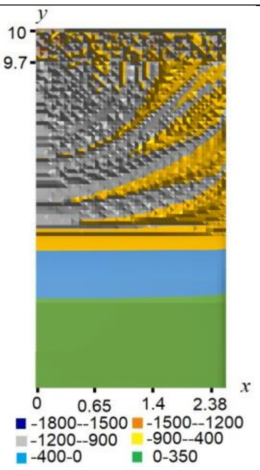

Fig. 27. Stress σ_{yy} when $h = h_2$, $t = t_3$

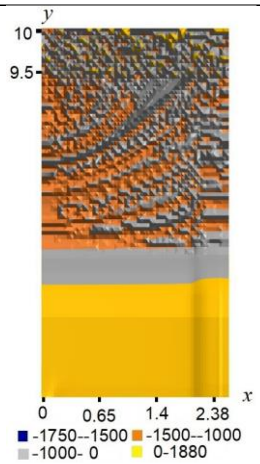

Fig. 28. Stress σ_{yy} when $h = h_3$, $t = t_3$

The radius of propagation of the summary plastic deformations from the centre of the contact zone with the indenter in the upper layer (Figs. 2 – 4) at the moment of time $t = t_1$ does not depend on the thickness of the upper metal layer.

From Figs. 5 – 7 it can be seen that at the moment of time $t = t_2$, if the metal layer is the thicker than the plastic deformations in it are the more intense and the plastic deformations in the glass layer in the area that borders the upper metal layer under the zone of contact between the base and the striker are smaller.

When $t = t_3$ (Figs. 8 – 10) the largest values of the Odquist parameter occur in the thinner upper layer. When $t \geq t_2$ (Figs. 11 – 19), the largest normal stresses in absolute value occur in the upper layer with the smaller thickness.

In the region of the lower boundary of the lower layer, linked to the rigid half-space, tensile stresses arise. This is due to the wave nature of the impact process and the rigid adhesion of the lower surface of the lower layer. Moreover, the greatest of these tensile stresses occur when the thickness of the upper layer is greater.

At the moment of time $t = t_1$ (Figs. 20 – 22) in the metal layer, the compressive normal stresses σ_{yy} are greater in the thicker layer. When $t \geq t_2$ (Figs. 23 – 28) the maximum compressive (maximum in absolute value) stresses σ_{yy} decrease with the thickness of the layer.

At moment $t = t_3$ in the area of the lower boundary of the lower glass layer, the greatest tensile stresses will increase with an increase in the thickness of the upper layer. Most likely, for this steel, it makes no sense to increase the thickness of the upper layer more than 0.5 mm.

CONCLUSIONS

The developed methodology of solving dynamic contact problems in an elastic-plastic dynamic mathematical formulation makes it possible to model the processes of impact, shock and non-stationary contact interaction with the elastic composite base more adequately. In this work, the process of impact on a two-layer base, consisting of an upper thin layer of metal and a lower main layer of glass, is adequately modelled. The fields of summary plastic deformations and normal stresses arising in the base are calculated depending on the thickness of top metal layer of the composite base. The results obtained make it possible to design new composite reinforced armed materials.

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Задача о плоском деформированном состоянии двухслойного тела в динамической упругопластической постановке (Часть II)

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Аннотация. Композитные материалы широко используются в промышленности и повседневной жизни. Математическое моделирование композитных материалов начинает активно разрабатываться в 50-х и 60-х годах прошлого века Композитные материалы начинают активно использоваться в промышленности только в конце 70-х годов прошлого столетия. С этого времени и по сей день интерес к композитным материалам не ослабевает, а запросы современной промышленности и производства все время увеличиваются. Расширяются области и отрасли применения композитных материалов. Для расчетов и разработки композитных материалов используется много различных методов. Данная статья есть часть вторая от предыдущей статьи, в которой рассматривается контактная задача о взаимодействии ударника с двухслойным композитным основанием в динамической упругопластической математической постановке. Это композитное основание жестко сцеплено с абсолютно твердым полупространством. Его первый (верхний) слой изготовлен из стали, а второй (нижний) – слой стекла. Стекло общедоступный дешевый аморфный материал, свойства которого не поддаются деградации в результате процессов старения, коррозии, ползучести. Слой стекла возможно усиливать за счет армирования и закалки. Поэтому композитные материалы, изготовленные на основе стекла, важны в современном производстве их использование дает большой экономический эффект. Предполагается жесткое сцепление слоев между собой. Процесс удара моделировался как нестационарная задача с равномерно распределенной нагрузкой в области контакта, изменяющейся по линейному закону. Были исследованы поля параметра Одквиста и нормальных напряжений в зависимости от размера области контакта. В отличие от предыдущей статьи (Часть I) в данной статье исследуется напряженно-деформированное состояние, поля параметра Одквиста и нормальных напряжений в зависимости от толщины первого (верхнего) слоя.

Ключевые слова: Плоская деформация, удар, композитные материалы, армированные материалы, бронированные материалы, упругопластическая, деформация.